Solving the Multiple Competitive Facilities Location Problem*

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Abstract

In this paper we propose five heuristic procedures for the solution of the multiple competitive facilities location problem. A franchise of several facilities is to be located in a trade area where competing facilities already exist. The objective is to maximize the market share captured by the franchise as a whole. We perform extensive computational tests and conclude that a two-step heuristic procedure combining simulated annealing and an ascent algorithm provides the best solutions.

Key words: Competitive facility location; Multiple facilities; Heuristic algorithms.

Introduction

The competitive multifacility location problem is similar in many ways to the p-median or p-center problems (Daskin, 1995). In the p-median or p-center models the objective is to minimize the cost for customers. In the competitive facility location model facilities compete against one another attempting to attract as many customers as possible in order to maximize market share. In p-median and p-center problems there are no existing facilities in the area whereas in the context of competitive facility location

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existing facilities compete. This resembles the conditional p-median or p-center models (Berman and Simchi-Levi, 1990; Chen, 1988; Chen and Handler, 1993; Drezner, 1989; Minieka, 1980) where facilities already exist in the area and one adds *p* new facilities such that the total cost is minimized. In the p-median, p-center, conditional p-median, or conditional p-center problems it is assumed that customers will use the service of the closest facility whether new or existing. Many competitive facility location problems also assume that customers patronize the closest facility.

The competitive facility location problem was first introduced by Hotelling (1929) who considered competition on a segment (such as "Main Street"). Hakimi (1983, 1986, 1990) formulated these problems on a network. Drezner (1982) solved the single facility location problem in the plane. These formulations are based on the assumption that a customer patronizes the closest facility.

A more realistic approach was introduced by Huff (1964, 1966) who suggested that customers divide their patronage among the competing facilities according to a gravity-based formula. The gravity-based formula suggests that the probability a customer will patronize a facility is proportional to the facility attractiveness and inversely proportional to the distance from it. Huff (1964, 1966) used the square footage of the facility as its attractiveness. Nakanishi and Cooper (1974) elaborated on Huff's model by suggesting techniques to estimate the attractiveness of facilities. Their approach is called the MCI model. More recent extensions to the Huff's model can be found in Jain and Mahajan (1979) and Bell et al. (1998). In our paper we assume that the attractiveness of the facilities is known. Drezner (1994) solved the single competitive facility location problem based on the gravity model, and Drezner (1998) solved its multiple version. Both these papers employ ascent algorithms.

In this paper we investigate heuristic solution methods for the multiple competitive facility location problem based on the gravity model. Five heuristic procedures are suggested for its solution and are compared for efficiency. The first is the steepest ascent approach suggested in Drezner (1998). The other four heuristics suggest new approaches to solving such problems. In the second heuristic we attempt to select non-random starting solutions for the ascent heuristic rather than employ randomly generated starting solutions. Such starting solutions should be (i) sparsely distributed and (ii) dissimilar to configurations encountered in previous solutions. The other three heuristics are based on discretization of the solution space. We observe that the objective function is more sensitive to the spatial configuration of the solution than to the exact location of each facility. For these three heuristic procedures we select a set of possible sites that cover the trade area and solve the problem restricted to this set. These are discrete heuristics. The third heuristic is an ascent heuristic on this set and the fourth one is a simulated annealing procedure. The fifth heuristic uses the solutions obtained by the previous two discrete procedures (since the simulated annealing performed better than the ascent algorithm it was selected for the fifth heuristic) as starting points for the continuous ascent algorithm (Drezner, 1998). We first find the spatial configuration and then refine the locations in the continuous space. The last four heuristics perform better than the simple ascent algorithm suggested in Drezner (1998). In the computational experiments the last two-phase approach (the fifth heuristic) provided the best solutions.

The Competitive Multifacility Location Problem

Consider a trade area with several already existing retail facilities competing for customers. The most commonly used competitive model for estimating market share is based on the gravity model. It is proposed that the probability that a customer selects a certain facility is proportional to its attractiveness and inversely proportional to a power of the distance to that facility. This rule defines for each customer a probability distribution of patronage for all the facilities in the area. Once this probability distribution is known, the market share of each facility can be evaluated by a summation over all the customers in the area.

Such a calculation is impractical for individual customers. Therefore, we normally define communities (such as cities, zip codes, Census tracts) and perform the calculation for each community rather than for individual customers. Let us have *n* communities in the trade area, each with a total discretionary income (buying power) b_i for $i = 1,\cdots,n$. There exist *k* competing facilities in the area, each with an attractiveness level of A_j for $j = 1,\cdots,k$. The distance between community *i* and facility *j* is d_{ij}. The power to which the distance is raised is λ. The market share, M_j, captured by facility *j* is determined according to the gravity model:

$$M_j = \sum_{i=1}^{n} b_i \frac{\frac{A_j}{d_{ij}^{\lambda}}}{\sum_{r=1}^{k} \frac{A_r}{d_{ir}^{\lambda}}} \tag{1}$$

Suppose that *p* new facilities are planned by a franchise in the area. The locations for such new facilities that maximize the total market share captured by the franchise are sought. Let these unknown locations be $X = X_1, \cdots X_p$. The attractiveness levels of the new facilities are $A_1^N, \cdots, A_p^N$. The distance between community *i* and new facility *s* is $d_i(X_s)$. It is possible that some of the existing facilities already belong to the incoming franchise. We sort the indices of the existing facilities such that the first *c* existing facilities belong to the incoming franchise (if no such facilities exist then *c*=0). The objective function *M(X)* of maximizing total market share captured by the incoming franchise (for both existing and new facilities) is:

(2) $$M(X_1, \cdots, X_p) = \sum_{i=1}^{n} b_i \frac{\sum_{r=1}^{c} \frac{A_r}{d_{ir}^{\lambda}} + \sum_{s=1}^{p} \frac{A_s^N}{d_i^{\lambda}(X_s)}}{\sum_{r=1}^{k} \frac{A_r}{d_{ir}^{\lambda}} + \sum_{s=1}^{p} \frac{A_s^N}{d_i^{\lambda}(X_s)}}$$

The terms in the numerator of equation (2) are proportional to the probabilities that a customer located at community *i* will patronize existing and new facilities of the incoming franchise and the terms in the denominator of (2) represent these values for all facilities in the trade area regardless of ownership. Problem (2) is very difficult to solve because many local optima may exist.

In this paper we employ a distance correction factor (see Appendix A) that models the distance between customers and facilities more accurately. A side benefit of the distance correction is that the number of local optima is reduced. We solved the problem in Drezner (1994) by the descent algorithm 100,000 times both for the problem without the distance correction and the problem with it. The numbers of the encountered local minima (which constitute lower bounds for the number of local minima) were 11, 470, 23714 for one, two, and three facilities, respectively. For four or more facilities we obtained more than 90,000 different local minima (out of 100,000 runs). The number of local minima when a distance correction was applied was lower, but still very large for larger values of p. The numbers of local minima encountered were 2, 3, 76, 1895, 17019, 65277 for p=1,...,6, respectively. For seven or more facilities the number of encountered local minima exceeded 90,000. Therefore, the problem modeled with the distance correction is somewhat easier to solve.

The Heuristic Approaches

We consider the following heuristics as summarized in Table 1:

Table 1: The Five Heuristic Procedures

Heuristic	Description
H-1	Steepest ascent (Drezner, 1998) starting from 100 randomly generated starting solutions and selecting the best final solution.
H-2	Steepest ascent as in H-1 but the 100 randomly generated starting solutions are generated in (i) sparse configurations and (ii) considering the previous starting and final solutions obtained so far (referred to as "history").
H-3	Selecting a grid of points that covers the trade area and applying a steepest ascent algorithm when the location of the new facilities is restricted to grid points.
H-4	Applying a simulated annealing approach for locations restricted to grid points.
H-5	Using the final solutions of H-4 (one can also use H-3) as starting solutions for the ascent algorithm H-1.

Heuristic H-1

The algorithm can be termed a "Generalized Weiszfeld Algorithm" (See Weiszfeld, 1936; Love et al., 1988; Drezner and Drezner, 1998). We find a local maximum by equating to zero the derivatives of (2) by each of the variables (incoming franchise locations' coordinates). This is done iteratively. The details of this Generalized Weiszfeld Algorithm are given in Drezner (1998).

Heuristic H-2

We consider two types of selection criteria for starting solutions: historical criterion and sparsity criterion.

1. Historical: It is plausible that experience from previous experiments can be used in selecting the starting solutions for subsequent experiments. Starting solutions, which are "similar", to previously used starting solutions or previous final solutions should be avoided because they are likely to lead to solutions that were already encountered. Being close to a solution that was previously encountered during the iterative procedure should be avoided because it is likely to lead to the same final solution already obtained. However, considering all intermediate solutions requires extensive computational effort that is not justified. Therefore, we only considered similarity to starting and final solutions. Also, eliminating starting solutions that are similar to previously encountered solutions, tends to create a better coverage of the solution space. A more successful variation on the same theme is to avoid similarity to bad solutions only. That is, the average value of the final objective function for previous experiments is calculated, and dissimilarity is checked only with starting and final solutions for which the final solution is below average.

The idea of considering past information is also used in diversification schemes within tabu search methods. For instance when solving the QAP, Kelly et al (1994) put forward a recency-based diversification (a new solution has to be distantly separated from the recent minimum in terms of number of moves and has to be non inferior in quality) and a frequency-based (where the frequency of occurrence of those attributes present in the previous solutions is taken into account by penalizing them from participating in subsequent moves etc). Also Thomas and Salhi (1998), when solving the constrained project scheduling, designed frequency-based diversifications combined with restricting some of the moves arising from those solutions. Though our approach considers historical information, as this is a powerful concept, it differs in many ways. For instance, our approach is not a frequency-based as it does not rely only on those frequent attributes but uses all previous solutions. It is also not recency-based approach. We generate the new solution using all previous solutions and not just the local minima. In addition, we measure the separation in a more analytical way, as will be explained later.

In order to operationalize this idea a "dissimilarity" measure between two solutions (which we also refer to as configurations) needs to be established. The following dissimilarity measure was adopted: Two configurations are "similar" if we can find a one-to-one correspondence between points in the first configuration and nearby points in the second configuration. We construct a matrix of p rows (representing points in the first configurations) and p columns (representing points in the second configuration). Each entry in the matrix is the distance (we used the square of the Euclidean distance) between the points in the two configurations. If the configurations are "perfectly similar", i.e., for every point in the first configuration there is a counterpart in the second configuration at exactly the same location, there will be a one-to-one assignment of a row to a column with all zero entries. Surely, one would not expect to have two identical configurations. Therefore, our objective is to find a one-to-one assignment between the points in the first configuration and the points in the second configuration such that the distances between the assigned points are small. This means finding a one-to-one assignment that minimizes the sum of the entries in the matrix. This is the simple assignment problem. As a measure of dissimilarity we used the solution to the assignment problem that minimizes the sum of the distances between points in the first configuration and their counterparts in the second configuration.

Solving assignment problems can be done quite efficiently by well-known methods (Bertsekas, 1991). Since we employ such an assignment problem many times and the value of p is relatively small, we used an approximate heuristic approach that is quicker than traditional methods. This simple assignment scheme proved to be efficient in empirical testing, see Appendix B for details of this heuristic procedure.

2. Sparsity: In general, good solutions consist of configurations where each facility captures part of the trade area. In such configurations new facilities tend to be far from one another to prevent cannibalization. It is therefore advantageous to select starting configurations that are “sparse”, i.e., each new facility is far from the other new facilities. The smallest distance between any two points of a configuration is used as the sparsity measure.

Implementation of Historical and Sparsity Measures:

1. Two numbers H (the number of randomly generated configurations for the history measure) and S (the number of configurations for the sparsity measures) are chosen.
2. S random configurations are created and the one with the highest sparsity is selected.
3. From the second iteration onward, Step 2 is repeated H times creating a set of H "sparse" solutions. The solution with the largest dissimilarity to historical data is selected as the next starting solution.
4. The problem is solved using the generalized Weiszfeld algorithm.
5. This is repeated K times (say K=100) and the best overall solution is selected.

<u>Heuristic H-3</u>

This heuristic is an ascent algorithm where the optimal solution is restricted to a set of points rather than the whole plane. A grid of possible locations for the facilities is selected. A square grid is simple to use but other grids can be used as well. For each point of the grid we define the set of neighboring points. In a square grid, each point (which is not on the boundary of the grid) has eight neighbors: four in the East, West, North, and South directions, and four in diagonal directions. For other configurations, the neighbors need to be clearly defined. For example, in hexagonal (beehive) configurations each point has a maximum of six neighbors.

In each iteration the p facilities, one at a time in a random order, are moved to each of the neighboring locations and the change in the value of the objective function is calculated. In a square grid there are less than $8p$ possible moves because a move outside the grid is not considered. Once an improvement in the objective function value is obtained, the move is performed and a new iteration begins. The algorithm stops when no improvement by such moves is encountered.

Heuristic H-4

Rather than using an ascent algorithm, we apply a simulated annealing approach (Kirkpatrick et al., 1983; Mavridou and Pardalos, 1997; Salhi, 1998) to solve the discrete problem defined by the grid in H-3. A simulated annealing procedure requires three parameters (also known as cooling schedules): the staring temperature, the rate at which the temperature is lowered α, and the limit on the number of iterations $K_{\max}$.

1. A starting temperature, T_0, and a starting random configuration on the grid is selected. Set the iteration number k=0.

2. A facility and its neighbor are both selected at random. This defines a random move resulting in a perturbed solution.
3. The change in the objective function value resulting from the random move is evaluated.
4. If the random move improves the value of the objective function, accept the perturbed solution and go to Step 6.
5. If the objective function value decreases by ΔF, compute $\delta = {}^{\Delta F}\!/_{T}$. The move is accepted with probability $e^{-\delta}$. If the move is not accepted, the current solution is not changed.
6. Set $T_{k+1}=\alpha T_k$ where T_k is the temperature control parameter at iteration k and $\alpha < 1$. If $k=K_{\max}$ stop and the best solution encountered throughout the procedure is selected as the solution. Otherwise, set $k=k+1$ and go to Step 2.

Heuristic H-5

The simulated annealing procedure H-4 is applied ten times and up to ten different final solutions are obtained. We use these final solutions as starting points for the generalized Weiszfeld algorithm. The generalized Weiszfeld algorithm serves as a post optimizer to fine-tune the solutions. One may apply the final solutions of H-3 rather than H-4. However, since H-4 was found superior to H-3 we opt to use H-4 for this heuristic.

Computational Experiments

All experiments were conducted using Microsoft FORTRAN codes that were run on a 300MHz Pentium computer. For all problems we used $\lambda=2$ and $c=0$. For most experiments we used either the *specific* problem given in Drezner (1994, 1995) or randomly generated problems. The randomly generated problems consist of 100 randomly generated communities in the unit square (each with a discretionary income of one), and seven randomly generated existing facilities in the same square (each with an attractiveness level of one). For the purpose of the distance correction an area of 0.01 was assigned to each community in all cases.

Determining the Parameters for H-2

We need to determine the number of draws (*S*) from which the most sparse configuration is selected along with the number of such draws (*H*) from which the one most dissimilar to previous configurations is selected. The total number of starting solutions drawn per iteration is *HS*. Note that *H*=*S*=1 is equivalent to H-1 because only one starting solution is randomly drawn. We tested H-2 for *p*=5, 8, and 10 new facilities. The attractiveness levels of the new facilities were set to 1/*p*, so that the total attractiveness level of the incoming franchise is 1. We tested 225 combinations of $1 \le H, S \le 15$. For each combination of *H* and *S* we randomly generated 10 such problems, and the value recorded for each problem is the best solution obtained from 100 starting solutions. This necessitated the solution of 675,000 problems and required about 80 hours of computer time. For each *H, S,* and *p* we calculated the average of the best results (out of 100 repetitions) for each of the ten problems. We plotted for each *p* a three-dimensional plot with the axes being "Historical" (*H*) and "Sparsity" (*S*) and the third dimension is the average of the ten best results. We found that the variance of the results is so large that no clear conclusion can be drawn. We therefore calculated for each pair *H* and *S* the average of up to 9 values for *H* or $H \pm 1$ and *S* or $S \pm 1$. This graph is more revealing and some trends can be observed. In Figure 1 we present the graph for *p*=8. It is still not clear which values of *H* and *S* are the best ones. We therefore averaged the results for all three *p*'s (*p*=5, 8, 10) and obtained the graphs depicted in Figures 2 and 3. Figure 2 depicts a 3-D view and Figure 3 depicts a view from the top. There is clearly a "mountain range" shaped like a crescent. We selected *H=7* and *S*=11 for our subsequent experiments.

Insert Figures 1, 2, and 3 about here

One may postulate that larger values of *H* and *S* must yield better starting solutions because both sparsity and historical considerations are more emphasized. However, we found that this is not the case. The sparsest configurations tend to be close to the periphery of the trade area, not necessarily good starting solutions. Solutions that are dissimilar to historical configurations tend to be either close to the periphery or very close to one another. A cluster of points yields a large sum of distances from other configurations. Therefore, having relatively small values for *S* and *H* tends to push the configuration towards these extremes (near the periphery or clustered) but provide configurations that are not too extreme.

Note that using *H*=*S*=1 (which is equivalent to H-1) is clearly inferior to employing sparsity and historical selection of starting solutions. We would also like to note that the *average* of the 100 experiments rather than the best one yields a "flat" graph showing no clear superiority for any *H* and *S.* This means that using sparse and dissimilar configurations as starting solutions does not improve the average quality of the solution. However, the best of 100 experiments of H-2 tends to be better than the best obtained by H-1 because the starting solutions cover the trade area better.

Insert Table 2 around here

Comparison between H-3 and H-4

Our next experiment tested H-3 and H-4 on ten randomly generated problems using a square grid of ten by ten points in a unit square. We first experimented with H-4 to determine the best parameters for the simulated annealing algorithm H-4. After extensive experiments we found that the following parameters produce the best results: A starting temperature of 1, the maximum number of iterations K_{max}=1000p iterations, and $\alpha = 1 - 5/K_{\max}$. Using this value of α reduces the temperature by a factor of $e^5 = 148$ yielding a proper final temperature. Since H-3 is about ten times faster than H-4, we compared the best results of 100 starting solutions for H-3 with 10 starting solutions for H-4. The results are summarized in Table 2. The simulated annealing H-4 provides better results than H-3 especially for larger values of p. Therefore we used the simulated annealing approach H-4 for generating starting solutions for H-5.

Experiments with the Three Heuristics H-1, H-2, and H-5

The heuristics H-3 and H-4 are markedly inferior to H-5 because H-5 improves on their results with very little extra computer time. Therefore, to establish the relative performance of the heuristic procedures it suffices to compare H-1, H-2, and H-5. Since H-5 is about ten times slower than H-1 and H-2, we compared 100 starting solutions for H-1 and H-2 with ten starting solutions for H-5.

Existing Test Problems: We first report the performance of these heuristics on the specific problem of Drezner (1994). The specific problem consists of 100 communities located on a ten by ten square grid with seven existing facilities. We employed the distance correction (see Appendix A) with an area of 0.01 for each community. The results are summarized in Table 3. The heuristic H-5 is definitely superior to the other two, with H-2 being second best.

Insert Tables 3, 4 and 5 around here

Randomly Generated Test Problems: We then tested the three heuristics on the ten randomly generated problems that were used for the comparison between H-1 and H-2. The results are presented in Table 4. Run times are for all starting solutions for each problem. Again, H-5 is clearly the best one with H-2 being second best.

Sensitivity Analysis: In order to test the sensitivity of the heuristics to the number of existing facilities we repeated the last experiment for different number of existing facilities: k=1, 2, 3, 4, 5, 10, 15, and 20. The results are reported in Table 5. The best results for each p and k are marked in boldface. It is interesting that the superiority of H-5 is manifested only for a larger number of existing facilities. We conclude that H-5 is best if the total attractiveness of the incoming franchise is significantly lower than the total attractiveness of the existing facilities. If the total attractiveness of the incoming franchise is comparable to the total attractiveness of the existing facilities, then H-1 and H-2 perform comparably well to H-5. Note that the average value for H-5 was never more than 0.03 below the best one (by H-1 or H-2) reported in Table 5. However, when H-5 is superior to the other two heuristics, it is often much better than H-1 or H-2 yielding a market share higher by up to 0.60.

Uniform Competition Test Problems: As a last experiment we tested a franchise moving into a new area which is surrounded by competition on the outside. To simulate such a situation we generated 40 existing facilities uniformly spaced on the periphery of the unit square, each with an attractiveness level of 1. For the incoming franchise we tested a total attractiveness of 1 (thus being dominated by the existing facilities) and a total attractiveness of 10. The results are summarized in Table 6. The best results (all obtained by H-5) are depicted in Figures 4 and 5. Since the best location for a single facility is at the center of the square, we did not present the solution for p=1 in these figures. H-5 again provided superior results when the total attractiveness is 1. All three heuristics provided almost identical solutions when the total attractiveness is set to 10. We conclude that the superiority of H-5 is more pronounced when the incoming franchise captures only a small portion of the market.

Insert Table 6 and Figures 4 and 5 around here

Figures 4 and 5 provide interesting configurations to be used for entering a new market that is surrounded by competition. One should repeat such an experiment for a real area that is not "exactly" a square to obtain useful market entry strategies.

Conclusions

In this paper we proposed four new heuristics for the solution of the multiple competitive location problem and compared them to an ascent algorithm on a set of test problems. Based on our set of test problems we conclude that the recommended heuristic is a combination of simulated annealing in discrete space (which is easier to implement than simulated annealing in continuous space) followed by an ascent algorithm in a continuous space.

As future research we propose to apply other metaheuristics such as tabu search, genetic algorithms, or continuous simulated annealing, rather than the discrete simulated annealing used in H-4. We also propose to use this procedure for analyzing various marketing entry strategies and evaluating the sensitivity of the solutions to the characteristics of the trade area. A first analysis of this kind is given in Figures 4 and 5.

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Appendix A: The Distance Correction

Each community consists of many customers and cannot be assumed a mathematical point. The distance between customers residing in a particular community and a facility vary from customer to customer because customers live in different locations within the community. Therefore, there is a need for some averaging of these distances. This issue is discussed in Drezner and Drezner (1997) where it is suggested to correct the distance by adding 24% of the area of the community to the square of the distance. This means to replace the distance d by $\sqrt{d^2 + 0.24A}$ where A is the area of the community.

Appendix B: Heuristic Solution of the Assignment Problem

Solving assignment problems can be done quite efficiently by well-known methods (Bertsekas, 1991). Since we employ such an assignment problem many times and the value of p is relatively small, we used a heuristic approach that is quicker than traditional methods. We found empirically that the proposed heuristic, H-A, finds the optimal solution more than 99% of the time for $p \leq 10$ (see Table B1). Also note that having the optimal solution of the assignment problem is not essential for the success of H-2 as long as a "wrong" solution is encountered very rarely. The heuristic H-A consists of two steps. We first apply the VAM method (Lawrence and Pasternack, 1998) that is designed for the transportation problem (of which the assignment problem is a special case) in order to obtain an initial solution. This means finding for each row and column the difference between the smallest entry in that row/column and the second smallest entry. The row/column with the maximal difference is selected and the smallest entry in that row/column is selected for an assignment. The row and column of the selected assignment are deleted from the matrix, and the process is repeated p times until a complete assignment is obtained. The final assignment is the VAM solution. As a second phase of H-A we check all $p(p-1)/2$ pair-wise exchanges of assignments. If an improvement is found, the change is accepted. We keep iterating until no pair-wise exchange improves the objective function. This heuristic procedure is very quick and for a small p nearly always finds the optimal solution. The VAM procedure without the second phase finds an optimal solution in fewer cases (see Table B1). For example, for p=10 the VAM approach found the optimal solution in 66.5% of cases while H-A found the optimal solution in 99.81% of cases. Run times of H-A for p=10 is 12 seconds for 10000 problems or 0.0012 seconds per assignment problem.

Table B1: Heuristic Solutions to the Assignment Problem

p	Optimal (Out of 10000)		Time (sec.)
	VAM	H-A	
2	10000	10000	0.49
3	10000	10000	0.92
4	9892	10000	1.55
5	9613	10000	2.52
6	9144	10000	3.76
7	8578	10000	5.27
8	7979	9998	7.10
9	7266	9990	9.36
10	6650	9981	12.08
11	5810	9959	15.18
12	5137	9926	18.82
13	4428	9899	23.06
14	3877	9852	27.86
15	3400	9811	33.32
16	2896	9735	39.22
17	2413	9692	46.07
18	2049	9570	53.51
19	1762	9507	61.69
20	1414	9426	70.84

Figure 1: Results for p=8

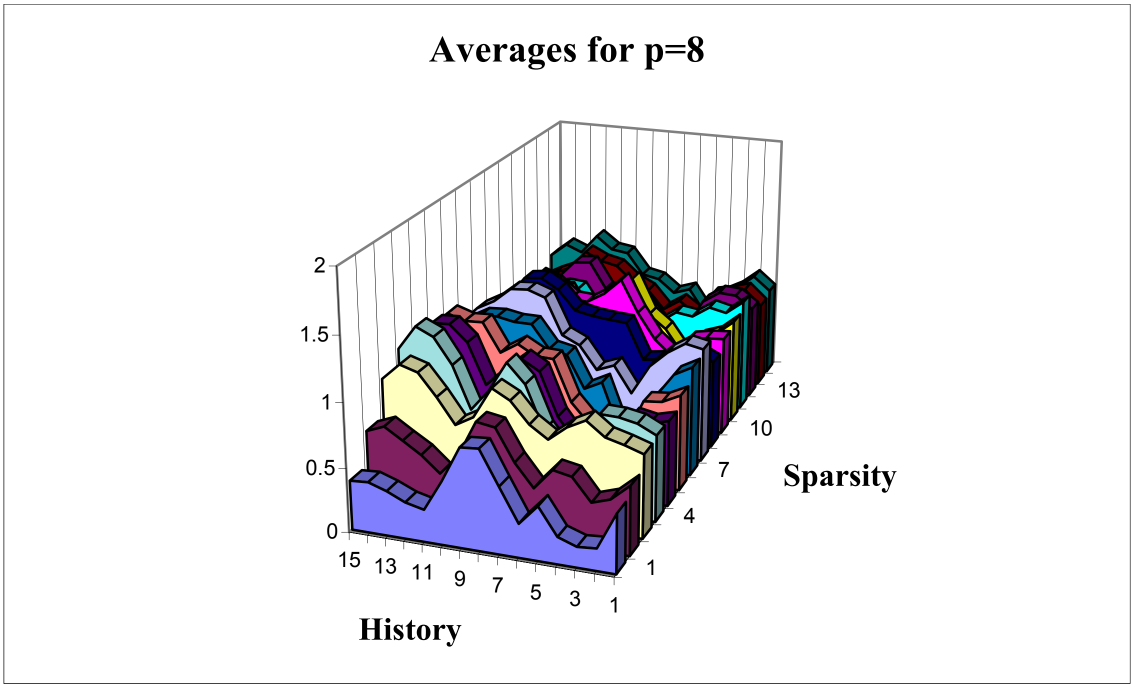

Figure 2: Sensitivity to Sparsity and Historical Parameters (3D view)

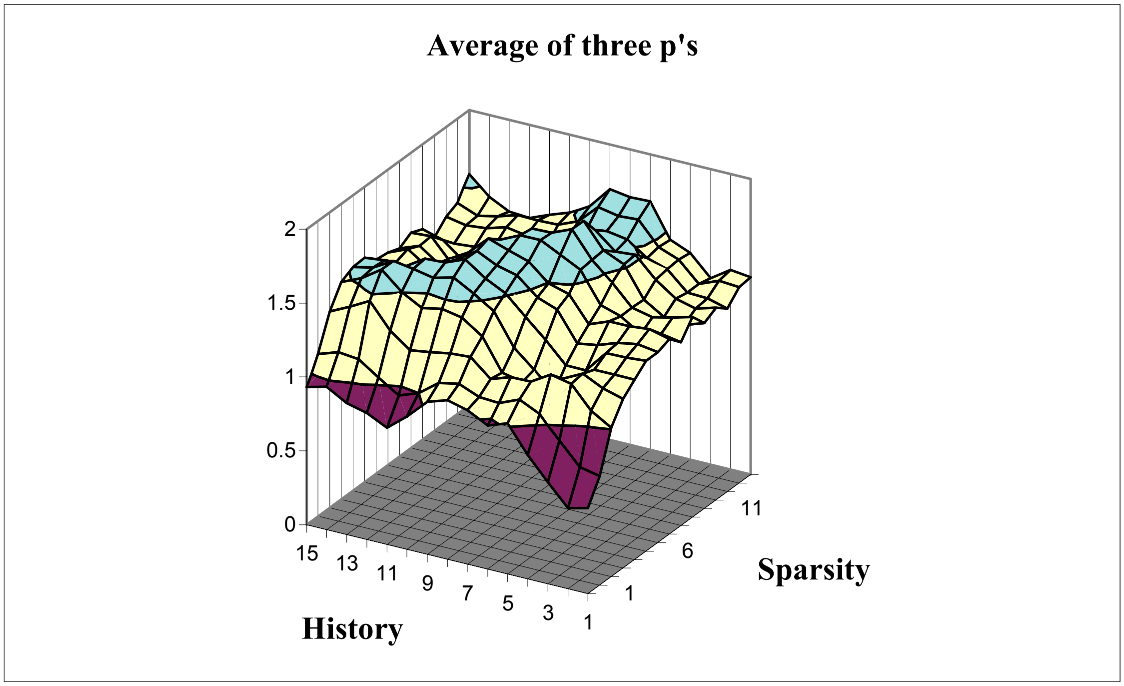

Figure 3: Sensitivity to Sparsity and Historical Parameters (top view)

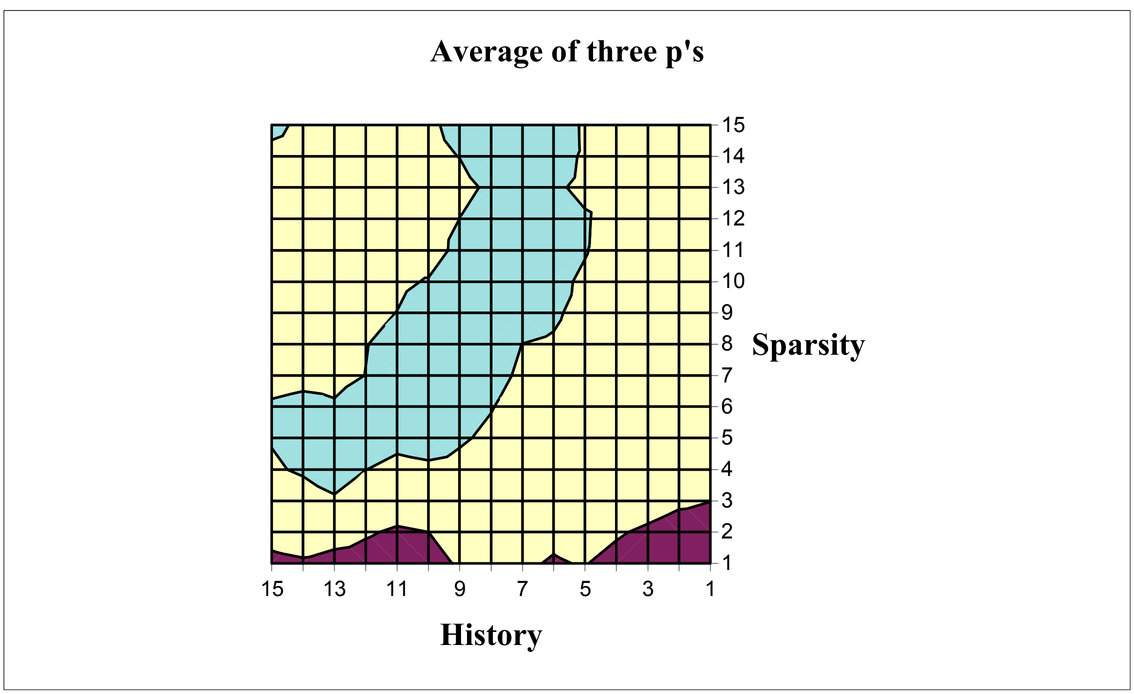

Table 2: Comparison between H-3 and H-4 on Ten Randomly Generated Problems

p	Market Share		Time (sec.) per Problem	
	H-3	H-4	H-3	H-4
1	16.82	16.82	0.4	1.1
2	20.03	20.03	1.2	2.8
3	21.21	21.21	2.7	5.2
4	21.89	21.89	5.1	8.2
5	22.43	22.43	8.5	11.9
6	22.76	22.79	12.8	16.2
7	22.96	22.97	18.7	21.2
8	23.10	23.14	25.4	26.9
9	23.24	23.27	33.5	33.2
10	23.32	23.37	44.8	40.5

Table 3: Results for the Specific Problem

p	Market Share			Time (sec.)		
	H-1	H-2	H-5	H-1	H-2	H-5
1	12.96	12.96	12.96	3.7	2.9	1.2
2	14.77	14.77	14.77	11.3	7.4	3.0
3	15.72	15.72	15.72	16.5	15.6	5.4
4	16.16	16.16	16.16	16.9	21.1	8.6
5	16.52	16.53	16.55	19.5	23.9	12.2
6	16.68	16.78	16.82	22.7	29.1	16.6
7	16.80	16.90	17.04	25.4	30.7	21.6
8	16.88	16.96	17.19	29.9	38.9	27.4
9	17.04	16.96	17.29	33.9	43.5	34.1
10	17.06	17.08	17.35	40.7	58.2	42.0

Table 4: Results of Ten Problems in Continuous Space

p	Market Share			Time (sec.) per Problem		
	H-1	H-2	H-5	H-1	H-2	H-5
1	16.94	16.94	16.94	1.4	1.6	1.1
2	20.34	20.34	20.34	3.4	4.2	2.9
3	21.78	21.78	21.78	6.6	7.9	5.3
4	22.55	22.61	22.62	9.5	12.0	8.5
5	23.14	23.21	23.29	14.7	17.9	12.7
6	23.42	23.59	23.71	20.2	26.1	17.2
7	23.45	23.68	23.98	27.1	34.2	23.3
8	23.91	23.92	24.15	35.5	44.8	30.6
9	23.95	23.96	24.26	48.1	57.1	41.2
10	23.84	24.00	24.43	56.8	72.8	50.2

Table 5: Sensitivity of the Market Share to the Number of Existing Facilities

p	**k=1**			**k=2**			**k=3**			**k=4**		
	H-1	**H-2**	**H-5**	**H-1**	**H-2**	**H-5**	**H-1**	**H-2**	**H-5**	**H=1**	**H=2**	**H-5**
1	**60.68**	**60.68**	**60.68**	**42.09**	**42.09**	**42.09**	**33.49**	**33.49**	**33.49**	**26.95**	**26.95**	**26.95**
2	**63.75**	**63.75**	**63.75**	**46.25**	**46.25**	**46.25**	**37.05**	**37.05**	**37.05**	**30.57**	**30.57**	**30.57**
3	**66.39**	**66.39**	**66.39**	**48.81**	**48.81**	48.79	**39.25**	**39.25**	**39.25**	**32.63**	**32.63**	32.62
4	**68.03**	**68.03**	68.01	**50.60**	**50.60**	**50.60**	**40.79**	**40.79**	40.78	**34.03**	**34.03**	**34.03**
5	**69.31**	**69.31**	**69.31**	**51.66**	**51.66**	**51.66**	41.79	41.79	**41.82**	34.93	34.93	**34.95**
6	**70.17**	**70.17**	70.16	**52.46**	**52.46**	52.45	42.56	**42.59**	42.57	35.54	35.57	**35.58**
7	**70.74**	**70.74**	70.72	**53.18**	**53.18**	53.17	43.14	**43.17**	43.16	35.99	36.06	**36.10**
8	**71.16**	**71.16**	71.14	**53.67**	53.66	**53.67**	43.58	43.62	**43.64**	36.31	36.42	**36.51**
9	**71.51**	**71.51**	71.48	**54.14**	54.12	54.13	43.93	43.96	**44.02**	36.60	36.62	**36.78**
10	**71.81**	**71.81**	71.78	**54.44**	54.42	**54.44**	44.13	44.21	**44.28**	36.74	36.85	**36.96**
	k=5			**k=10**			**k=15**			**k=20**		
1	**22.04**	**22.04**	22.03	**13.12**	**13.12**	**13.12**	**9.89**	**9.89**	**9.89**	**7.85**	**7.85**	**7.85**
2	**25.90**	**25.90**	25.89	**15.43**	**15.43**	15.42	**11.38**	11.31	11.35	**8.84**	8.83	8.82
3	**27.81**	**27.81**	27.79	**16.57**	**16.57**	16.55	11.97	**12.05**	12.04	9.09	9.25	**9.27**
4	**28.87**	28.84	28.85	17.25	17.29	**17.30**	12.33	12.35	**12.36**	9.27	9.41	**9.49**
5	29.65	29.61	**29.69**	17.46	17.54	**17.66**	12.37	12.43	**12.59**	9.44	9.48	**9.66**
6	30.11	30.10	**30.19**	17.66	17.69	**17.91**	12.36	12.58	**12.77**	9.28	9.46	**9.74**
7	30.42	30.46	**30.61**	17.72	17.84	**18.07**	12.39	12.53	**12.85**	9.35	9.37	**9.80**
8	30.77	30.75	**30.92**	17.86	17.89	**18.17**	12.55	12.66	**12.97**	9.28	9.40	**9.86**
9	30.84	30.93	**31.16**	17.89	17.90	**18.30**	12.49	12.49	**13.04**	9.43	9.38	**9.89**
10	30.89	31.01	**31.34**	17.77	17.90	**18.38**	12.36	12.54	**13.05**	9.30	9.39	**9.90**

Table 6: Results in a Square Surrounded by Competition

p	Total Attractiveness =1			Total Attractiveness =10		
	H-1	H-2	H-5	H-1	H-2	H-5
1	6.64	6.64	6.64	28.65	28.65	28.65
2	7.04	7.03	7.04	30.74	30.74	30.74
3	7.27	7.27	7.29	32.32	32.32	32.32
4	7.39	7.39	7.46	33.42	33.42	33.42
5	7.24	7.21	7.52	33.88	33.88	33.88
6	7.27	7.22	7.58	34.28	34.28	34.28
7	7.34	7.20	7.63	34.64	34.64	34.64
8	7.32	7.30	7.67	34.93	34.93	34.93
9	7.41	7.25	7.68	35.18	35.18	35.18
10	7.40	7.30	7.70	35.30	35.30	35.31

Figure 4: Solutions in a Square Surrounded by Competition (Total Attractiveness =10)

Figure 5: Solutions in a Square Surrounded by Competition (Total Attractiveness =1)

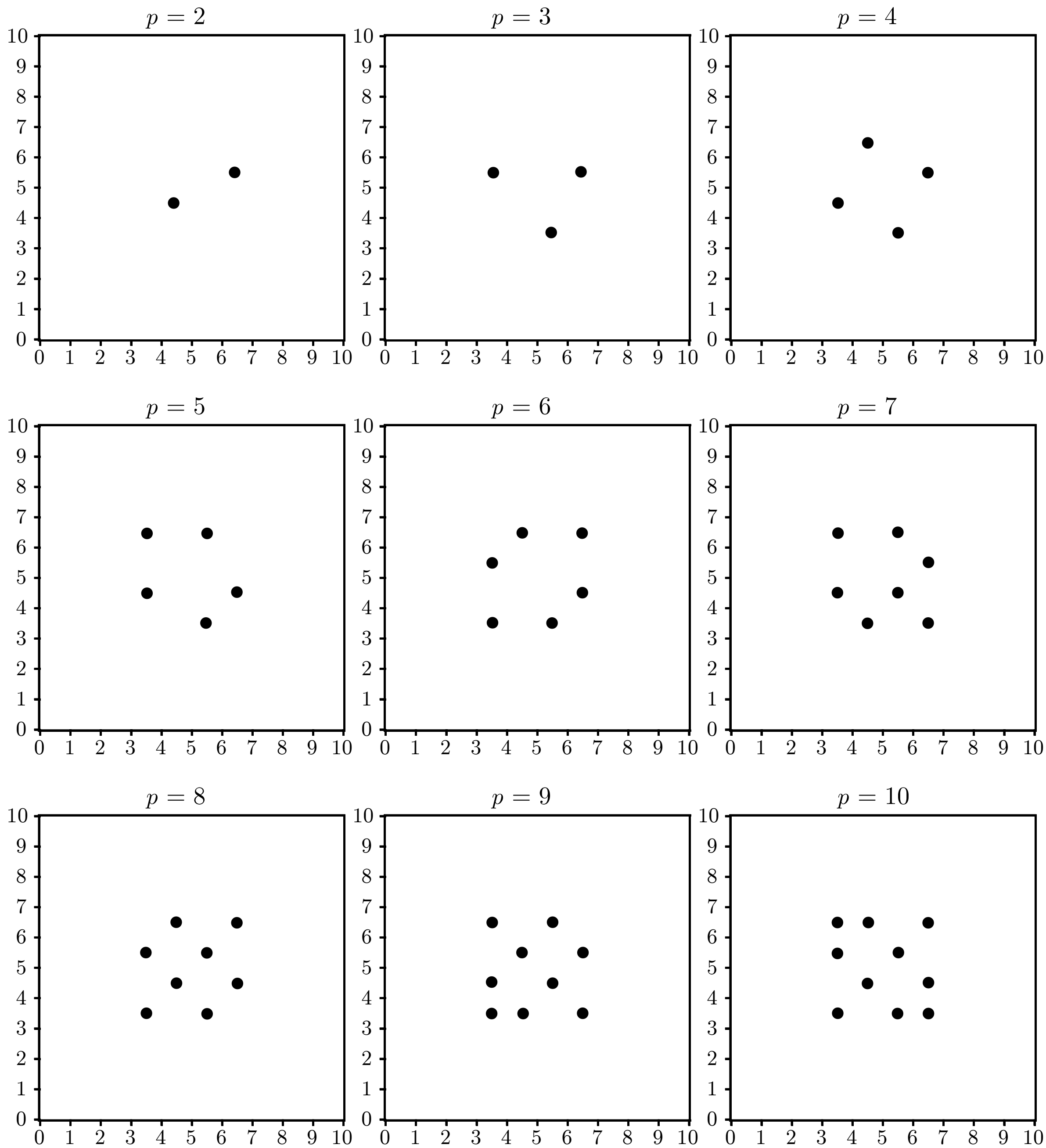

Figure 4

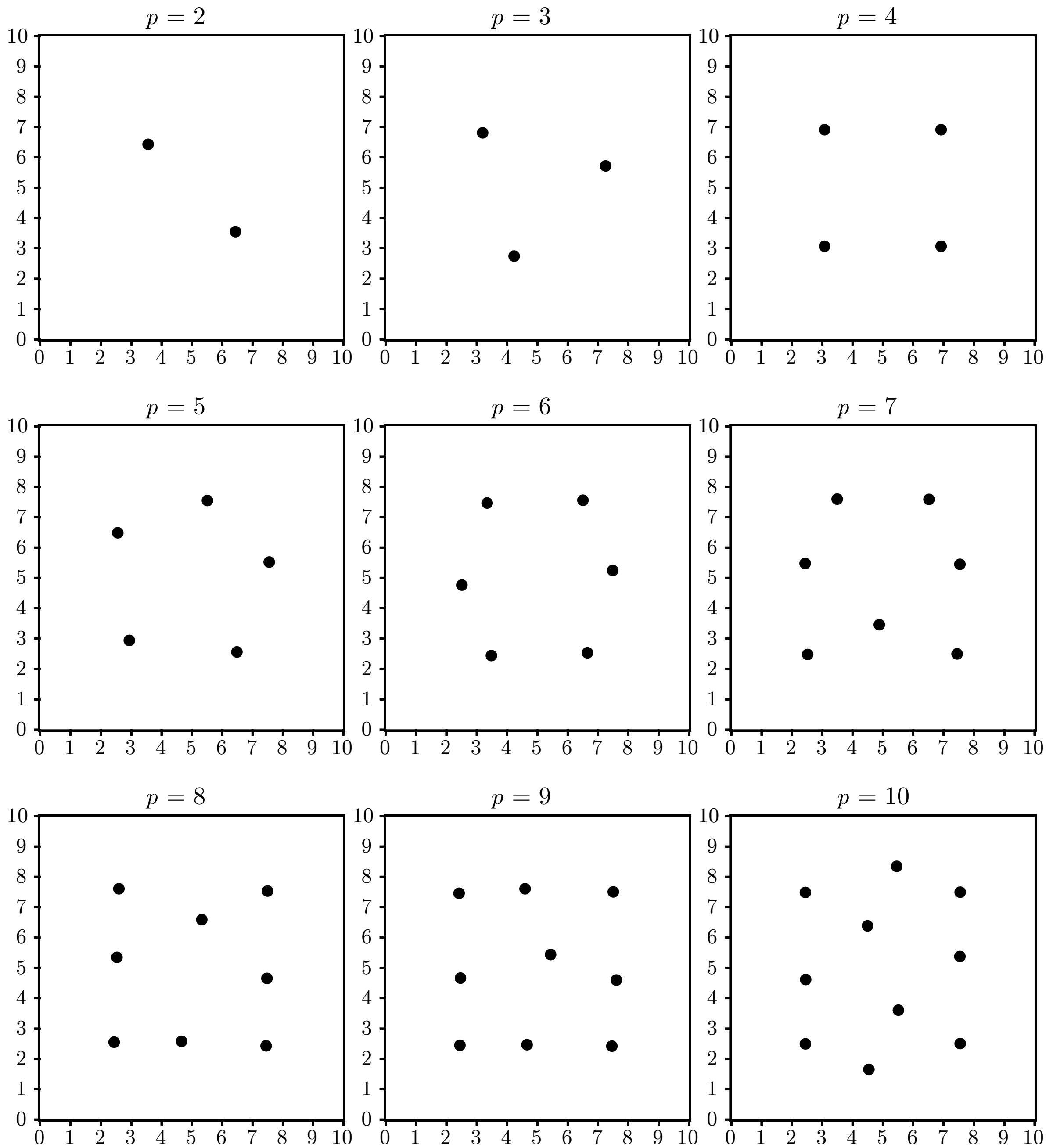

Figure 5